Indirect Proof and Inequalities in Two triangles (pp 302-304)

I can read and write indirect proofs.
I can state and apply the hinge theorem and its converse.

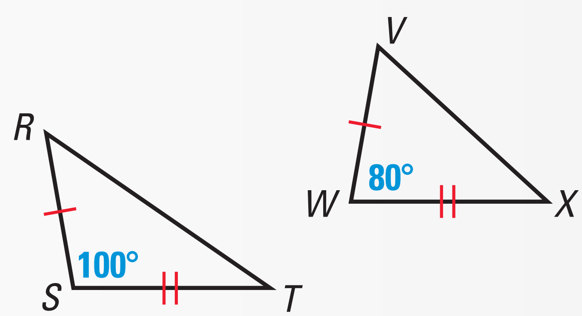
CONCEPT SUMMARY — GUIDELINES FOR WRITING AN INDIRECT PROOF

1. Identify the statement that you want to prove is true.
2. Begin by assuming the statement is false; assume its opposite is true.
3. Obtain statements that logically follow from your assumption.
4. If you obtain a contradiction, then the original statement must be true.

THEOREMS

THEOREM 5.14 ***Hinge Theorem***

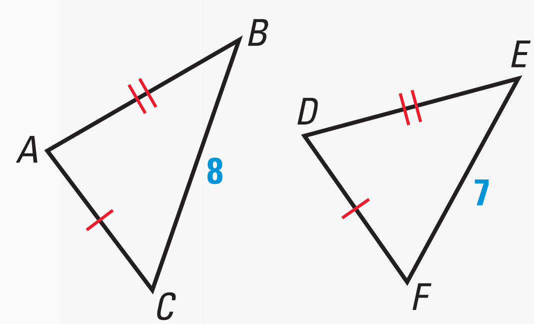
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$RT > VX$

THEOREM 5.15 ***Converse of the Hinge Theorem***

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



$m\angle A > m\angle D$

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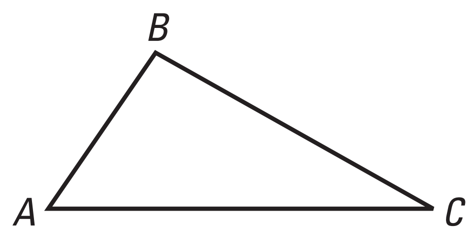
Up to now, all of the proofs in this textbook have used the Laws of Syllogism and Detachment to obtain conclusions directly. In this lesson, you will study *indirect proofs*. An **indirect proof** is a proof in which you prove that a statement is true by first assuming that its opposite is true. If this assumption leads to an impossibility, then you have proved that the original statement is true.

EXAMPLE 1 *Using Indirect Proof*

Use an indirect proof to prove that a triangle cannot have more than one obtuse angle.



SOLUTION

GIVEN ▶ $\triangle ABC$

PROVE ▶ $\triangle ABC$ does not have more than one obtuse angle.

Begin by assuming that $\triangle ABC$ *does* have more than one obtuse angle.

$m\angle A > 90°$ and $m\angle B > 90°$ — **Assume $\triangle ABC$ has two obtuse angles.**

$m\angle A + m\angle B > 180°$ — **Add the two given inequalities.**

You know, however, that the sum of the measures of all *three* angles is 180°.

$m\angle A + m\angle B + m\angle C = 180°$ — **Triangle Sum Theorem**

$m\angle A + m\angle B = 180° - m\angle C$ — **Subtraction property of equality**

So, you can substitute $180° - m\angle C$ for $m\angle A + m\angle B$ in $m\angle A + m\angle B > 180°$.

$180° - m\angle C > 180°$ — **Substitution property of equality**

$0° > m\angle C$ — **Simplify.**

The last statement is *not possible*; angle measures in triangles cannot be negative.

▶ So, you can conclude that the original assumption must be false. That is, $\triangle ABC$ cannot have more than one obtuse angle.

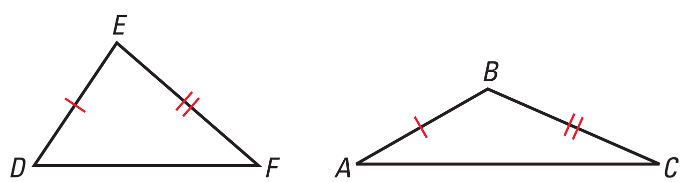
Indirect Proof and Inequalities in Two triangles (pp 302-304)

1. Use an indirect proof to prove that a triangle cannot have more than one right angle.

EXAMPLE 2 ***Indirect Proof of Theorem 5.15***

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$



SOLUTION Begin by assuming that $m\angle B \ngtr m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given information that $AC > DF$. So the original assumption that $m\angle B \ngtr m\angle E$ cannot be correct. Therefore, $m\angle B > m\angle E$.

STUDENT HELP

Study Tip
The symbol $\ngtr$ is read as "is not greater than."

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2. Write an indirect proof that through a point P on line t, there is at most one line perpendicular to line t.

EXAMPLE 3 *Finding Possible Side Lengths and Angle Measures*

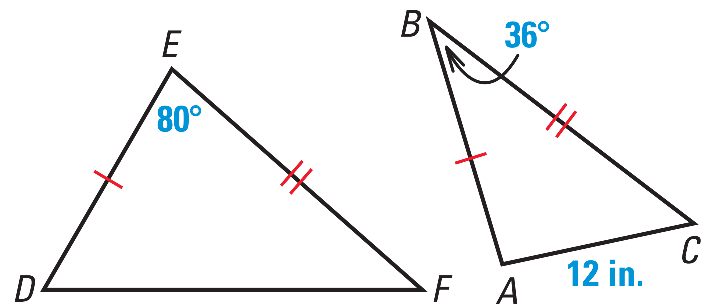
You can use the Hinge Theorem and its converse to choose possible side lengths or angle measures from a given list.

a. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $AC = 12$ inches, $m\angle B = 36°$, and $m\angle E = 80°$. Which of the following is a possible length for $\overline{DF}$: 8 in., 10 in., 12 in., or 23 in.?

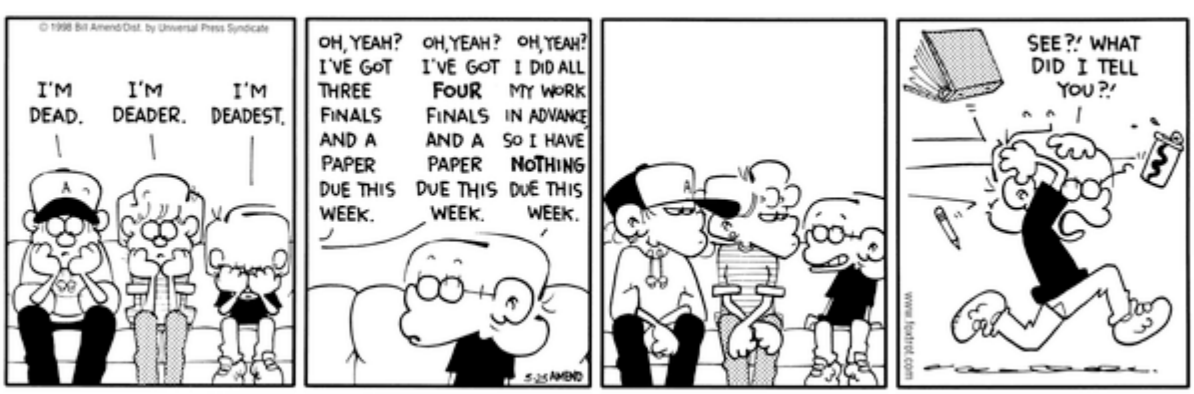
b. In a $\triangle RST$ and a $\triangle XYZ$, $\overline{RT} \cong \overline{XZ}$, $\overline{ST} \cong \overline{YZ}$, $RS = 3.7$ centimeters, $XY = 4.5$ centimeters, and $m\angle Z = 75°$. Which of the following is a possible measure for $\angle T$: 60°, 75°, 90°, or 105°?

SOLUTION

a. Because the included angle in $\triangle DEF$ is larger than the included angle in $\triangle ABC$, the third side $\overline{DF}$ must be longer than $\overline{AC}$. So, of the four choices, the only possible length for $\overline{DF}$ is 23 inches. A diagram of the triangles shows that this is plausible.



b. Because the third side in $\triangle RST$ is shorter than the third side in $\triangle XYZ$, the included angle $\angle T$ must be smaller than $\angle Z$. So, of the four choices, the only possible measure for $\angle T$ is 60°.



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3. _____ In $\triangle ABC$ & $\triangle DEF$, $\overline{AC} = \overline{DF}$, $\overline{BC} \cong \overline{EF}$, AB = 11 in, ED = 15 in, & $m\measuredangle F = 58°$. What is possible measure for $\measuredangle C$.

A. $45°$
B. $58°$
C. $80°$
D. $90°$

4. _____ In $\triangle GHI$ & $\triangle JKL$, $\overline{GH} \cong \overline{JK}$, $\overline{HI} \cong \overline{KL}$, GI = 9 cm, $m\measuredangle H = 45°$ & $m\measuredangle K = 65°$. Which of the following is possible for JL?

A. 5 cm
B. 7 cm
C. 9 cm
D. 11 cm

EXAMPLE 4 *Comparing Distances*

TRAVEL DISTANCES You and a friend are flying separate planes. You leave the airport and fly 120 miles due west. You then change direction and fly W 30° N for 70 miles. (W 30° N indicates a north-west direction that is 30° north of due west.) Your friend leaves the airport and flies 120 miles due east. She then changes direction and flies E 40° S for 70 miles. Each of you has flown 190 miles, but which plane is farther from the airport?

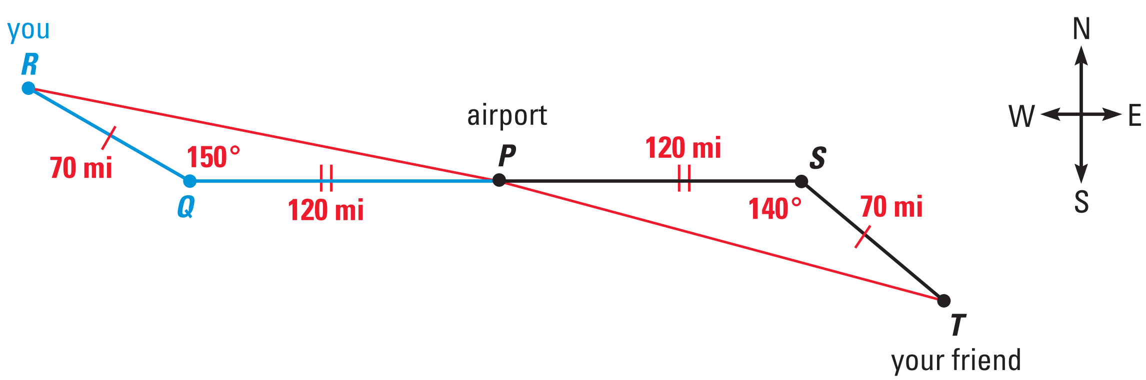
FOCUS ON CAREERS

REAL LIFE

AIR TRAFFIC CONTROLLERS help ensure the safety of airline passengers and crews by developing air traffic flight paths that keep planes a safe distance apart.

SOLUTION

Begin by drawing a diagram, as shown below. Your flight is represented by $\triangle PQR$ and your friend's flight is represented by $\triangle PST$.



Because these two triangles have two sides that are congruent, you can apply the Hinge Theorem to conclude that $\overline{RP}$ is longer than $\overline{TP}$.

▶ So, your plane is farther from the airport than your friend's plane.

Indirect Proof and Inequalities in Two triangles (pp 302-304)

5. You and a friend are flying separate planes. You leave the airport and fly 200 miles due north. You then change direction and fly W $60°$ N for 90 miles. Your friend leaves the airport and flies 200 miles due south. She then changes direction and flies E $15°$ S for 90 miles. Each of you has flown 290 miles, but which plane is further from the airport?

6. _____ In $\triangle RST$ & $\triangle XYZ, \overline{RT} \cong \overline{XZ}, \overline{ST} \cong \overline{YZ},$ RS = 24 cm, XY = 18 cm, & $m\measuredangle Z = 62°$, which of the following is a possible measure for $\measuredangle T$?

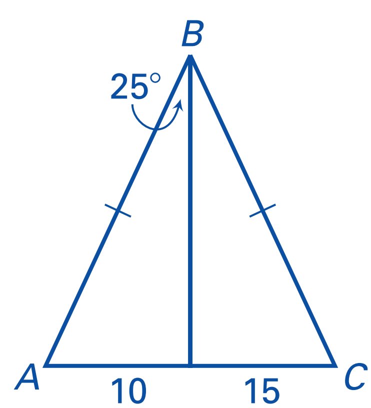
A. $34°$
B. $48°$
C. $56°$
D. $77°$

7. ______ In $\triangle ABC$ & $DEF, \overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF},$ AC = 14 in, $m\measuredangle B = 70°$, & $m\measuredangle E = 40°$. Which is a possible measurement for DF?

A. 8 in
B. 16 in
C. 20 in
D. 25 in

8. _____ Which of the following is not a possible measure for $\measuredangle DBC$?

A. $30°$
B. $27°$
C. $20°$
D. $31°$
E. $50°$



9. What is the first step in any indirect proof?